THE NONLINEAR REAL EXCHANGE RATE GROWTH MODEL

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ABSTRACT

Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy.
The basic aim of this paper is to provide a relatively simple chaotic real exchange rate growth model that is capable of generating stable equilibria, cycles, or chaos.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \left(\frac{b_m + \beta\, b - s}{b_m - b - (1/p)} \right)$ *plays a crucial role in explaining local stability of the real exchange rate growth , where, s – private saving rate ,* b_m *- marginal budget deficit coefficient, b – average budget deficit coefficient, p - productivity and n – net capital outflow rate, β – coefficient which describes relation between budget deficit and net capital outflow.*

***Keywords**: real exchange rate, appreciation, chaos, stability.*

Introduction:

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983) , Grandmont (1985), Goodwin (1990), Medio (1993,1996), Lorenz (1993), Jablanovic (2010, 2011, 2012), etc.
Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.
National saving is the source of the supply of loanable funds. Domestic investiment and net capital outflow are the sources of the demand for loanable funds. At the equilibrium interest rate, the amount that people want to save exactly balances the amount that people want to borrow for the purpose of buying domestic capital and foreign capital. At the equilibrium interest rate , the amount that people want to save exactly balances the desired quantities of domestic investment and net capital outflow.

The real exchange rate is determined by the supply and demand for foreign – currency exchange. The supply of domestic currency to be exchanged into foreign currency comes from net capital outflow. The demand for domestic currency comes from net exports. Because a lower real exchange rate stimulates net export, the demand curve is downward sloping. At the equilibrium real exchange rate, the demand for domestic currency to buy net export exactly balances the supply of domestic currency to be exchanged into foreign currency to buy assets abroad.
The supply and demand for loanable funds determine the real interest rate. The interest rate determine net capital outflow, which provides the supply of dollars in the market for foreign – currency exchange. The supply and demand for domestic currency in the market for foreign-currency exchange determine the real exchange rate.
In an open economy, government budget deficit raises real interest rates, crowds out domestic investment, decreases net capital outflow, causes the domestic currency to appreciate. However this appreciation makes domestic goods and services more expensive compared to foreign goods and services. In this case, exports fall, and imports rise. Namely, net exports fall. Rise in the budget deficit level causes appreciation of domestic currency. The trade balance is pushed toward deficit. The budget and trade deficits are called the twin deficits.

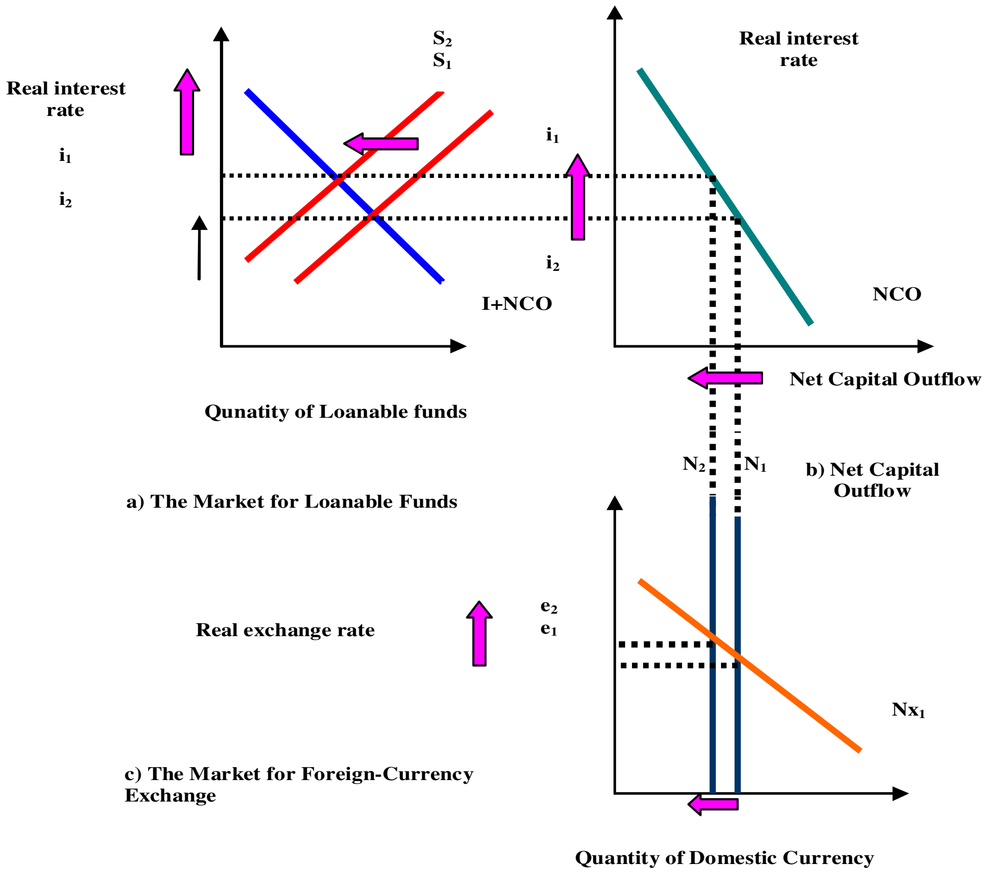

Figure 1: In an open economy, government budget deficit raises real interest rates, crowds out domestic investment, decreases net capital outflow, and causes the domestic currency to appreciate.

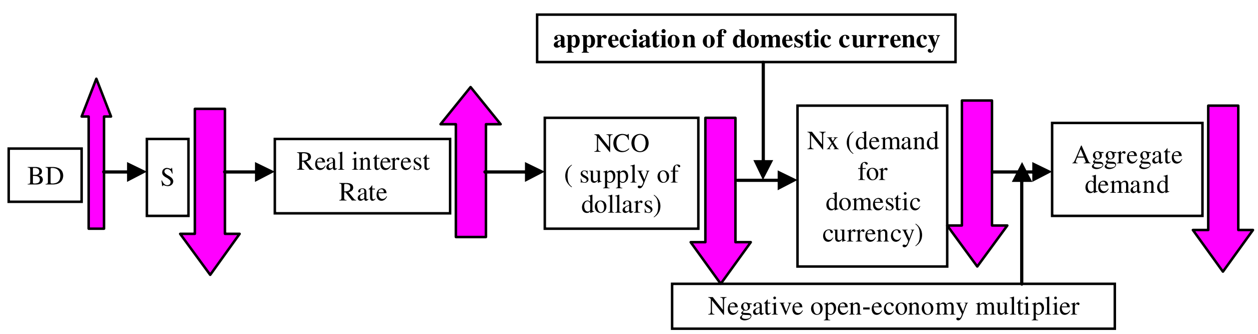

Figure 2: Relation between government budget deficit, national saving, real interest rate, net capital outflow, net export and aggregate demand (BD – budget deficit, S – national saving, NCO – net capital outflow, Nx- net export).

The Model:

The chaotic real exchange rate growth model is consisted :

$$b_m = \frac{\Delta Bd}{\Delta Y} \quad (1)$$

$$b = \frac{Bd}{Y} \quad (2)$$

$$S_t = S_{pt} - Bd_t \quad (3)$$

$$S_t = I_t + NCO_t \quad (4)$$

$$S_{pt} = s\,Y_t \quad (5)$$

$$I_t = \Delta L \quad (6)$$

$$Y_t = L_t^{1/2} \quad (7)$$

$$p = Y_t / L_t \quad (8)$$

$$NCO_t = \alpha - \beta\, Bd_t \quad (9)$$

$Nx_t = \gamma - \delta\, e_t$ (10)

Where Y – the real gross domestic product, I – investment, S – saving, Bd – budget deficit, L – labour, NCO – net capital outflow, Nx - net export. s – private saving rate, b_m – the marginal budget deficit coefficient, b – the average budget deficit coefficient, p- productivity, n- net capital outflow rate, e–real exchange rate, α , β ,γ ,δ – coefficients. (1) defines the marginal budget deficit coefficient ; (2) defines the average budget deficit coefficient; (3) defines national saving; (4) describes the identity between saving and investment plus net capital outflow; (5) determines the private saving; (6) determines investment as the increased labour; (7) contains the production function; and (8) defines productivity, (9) determines net capital outflow, and (10) defines the relation between net export and exchange rate.

It is supposed that α=0, and γ =0. By substitution one derives:

$$e_{t+1} = \frac{b_m + \beta\, b - s}{b_m - b - (1/p)} e_t - \frac{\delta}{\beta\, b\, [b_m - b - (1/p)]} e_t^2$$

(11)

Further, it is assumed that the current value of the real exchange rate is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the real exchange rate growth rate depends on the current size of the real exchange rate, e, relative to its maximal size in its time series e^m. We introduce ε as ε = e/e^m. Thus ε range between 0 and 1. Again we index ε by t, i.e., write ε_t to refer to the size at time steps t = 0,1,2,3,... Now growth rate of the real exchange rate is measured as

$$\varepsilon_{t+1} = \frac{b_m + \beta\, b - s}{b_m - b - (1/p)} \varepsilon_t - \frac{\delta}{\beta\, b\, [b_m - b - (1/p)]} \varepsilon_t^2 \quad (12)$$

This model given by equation (12) is called the logistic model. For most choices of β, δ, s, b_m , b, and p there is no explicit solution for (12). Namely, knowing β, δ, s, b_m , b, p, and measuring ε_0 would not suffice to predict ε_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.
This kind of difference equation (12) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of ε_t. This difference equation (12) will possess a chaotic region. Two properties of the chaotic solution are important : firstly, given a starting point ε_0 the solution is highly sensitive to variations of the parameters β , δ , s, b_m , b, and p ; secondly, given the parameters β, δ , s , b_m , b, and p, the solution is highly sensitive to variations of the initial point ε_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

The Logistic Equation:

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.
It is possible to show that iteration process for the logistic equation
$z_{t+1} = \pi z_t(1 - z_t),\ \pi \in [0,4],\ z_t \in [0,1]$ (13)
is equivalent to the iteration of growth model (12) when we use the identification:

$$z_t = \left[\frac{\delta}{\beta\, b\,(b_m + \beta\, b - s)}\right] \varepsilon_t \quad \text{and}$$

$$\pi = \left(\frac{b_m + \beta\, b - s}{b_m - b - (1/p)}\right) \quad (14)$$

Using (12) and (14) we obtain:

$$z_{t+1} = \left[\frac{\delta}{\beta\, b\,(b_m + \beta\, b - s)}\right] \varepsilon_{t+1}$$

$$= \left[\frac{\delta}{\beta\, b\,(b_m + \beta\, b - s)}\right] \left\{\left[\frac{b_m + \beta\, b - s}{b_m - b - (1/p)}\right] \varepsilon_t - \frac{\delta}{\beta\, b\,[b_m - b - (1/p)]} \varepsilon_t^2\right\}$$

$$= \left\{\frac{\delta}{\beta\, b\,[b_m - b - (1/p)]}\right\} \varepsilon_t - \left\{\frac{\delta^2}{\beta^2\, b^2\,(b_m + \beta\, b - s)[b_m - b - (1/p)]}\right\} \varepsilon_t^2$$

On the other hand, using (12) and (13) we obtain:

$z_{t+1} = \pi z_t(1 - z_t) =$

$$= \left(\frac{b_m + \beta\, b - s}{b_m - b - (1/p)}\right) \left[\frac{\delta}{\beta\, b\,(b_m + \beta\, b - s)}\right] \varepsilon_t \left\{1 - \left[\frac{\delta}{\beta\, b\,(b_m + \beta\, b - s)}\right] \varepsilon_t\right\}$$

$$= \left\{\frac{\delta}{\beta\, b\,[b_m - b - (1/p)]}\right\} \varepsilon_t - \left\{\frac{\delta^2}{\beta^2\, b^2\,(b_m + \beta\, b - s)[b_m - b - (1/p)]}\right\} \varepsilon_t^2$$

Thus we have that iterating $\varepsilon_{t+1} = \frac{b_m + \beta\, b - s}{b_m - b - (1/p)} \varepsilon_t - \frac{\delta}{\beta\, b\,[b_m - b - (1/p)]} \varepsilon_t^2$ is really the same as iterating $z_{t+1} = \pi z_t(1 - z_t)$ using $z_t = \left[\frac{\delta}{\beta\, b\,(b_m + \beta\, b - s)}\right] \varepsilon_t$ and $\pi = \left(\frac{b_m + \beta\, b - s}{b_m - b - (1/p)}\right)$.
It is important because the dynamic properties of the logistic equation (13) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :

(i) For parameter values $0 < \pi < 1$ all solutions will converge to z = 0;
(ii) For $1 < \pi < 3, 57$ there exist fixed points the number of which depends on π;
(iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$;
(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
(v) For $3 < \pi < 4$ all solutions will continously fluctuate;
(vi) For $3,57 < \pi < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Empirical Evidence:

The main aim of this paper is to analyze the real exchange rate stability of the U.S. dollar in the period 2004-2011. by using the presented non-linear, logistic real exchange rate growth model (12) :

$$\varepsilon_{t+1} = \pi \varepsilon_t - \upsilon \varepsilon_t^2 \qquad (15)$$

where ε – real exchange rate , $\pi = \left(\frac{b_m + \beta b - s}{b_m - b - (1/p)} \right)$ and

$$\upsilon = \frac{\delta}{\beta b \left[b_m - b - (1/p) \right]}$$

Firstly, data on the real exchange rate are transformed (www.imf.org) from 0 to 1, according to our supposition that actual value of the real exchange rate, e , is restricted by its highest value in the time-series, e^m. Further , we obtain time-series of $\varepsilon = e / e^m$.

Table 1: The estimated model (15): Real exchange rate: U.S. dollar, 2004-2011. (www.imf.org) (R=0.54262 Variance explained 29.444%)

		π	υ
US dollar	**Estimate**	1.313685	.358433
	Std. Err.	.426769	.451410
	t(5)	3.078213	0.794031
	p-level	0.027525	0.463180

Conclusion:

This paper suggests conclusion for the use of the chaotic real exchange rate growth model in predicting the fluctuations of the real exchange rate. The chaotic model (12) has to rely on specified parameters β , δ , s , b_m , b, and p, and initial value of the real exchange rate, ε_0. But even slight deviations from the values of parameters: β , δ , s , b_m , b, and p and initial value of the real exchange rate , ε_0 show the difficulty of predicting a long-term exchange rate behaviour.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \left(\frac{b_m + \beta b - s}{b_m - b - (1/p)} \right)$ plays a crucial role in explaining local stability of the real exchange rate growth , where, s – private saving rate , b_m - marginal budget deficit coefficient, b – average budget deficit coefficient, p-productivity and n – net capital outflow rate, β – coefficient which describes relation between budget deficit and net capital outflow.

An estimated values of the coefficient π were greater than 1. This result confirms low and stable U.S. dollar appreciation in the observed period. To stabilize exchange rate, it is important to decrease the budget deficit, to increase private saving, and to increase productivity.

References:

[1] Benhabib, J., & Day, R. H. (1981). Rational choice and erratic behaviour. *Review of Economic Studies, 48*. 459-471.
[2] Benhabib, J., & Day, R. H. (1982). Characterization of erratic dynamics in the overlapping generation model. *Journal of Economic Dynamics and Control, 4,* 37-55.
[3] Day, R. H. (1982). Irregular growth cycles. *American Economic Review, 72,* 406-414.
[4] Day, R. H. (1983). The emergence of chaos from classica economic growth. *Quarterly Journal of Economics, 98,* pg. 200-213.
[5] Goodwin, R. M. (1990). *Chaotic economic dynamics*. Oxford: Clarendon Press
[6] Grandmont, J. M. (1985). On enodgenous competitive business cycles. *Econometrica, 53,* pg. 994-1045
[7] Jablanović, V. (2010). *Chaotic population growth.* Belgrade: Cigoja.
[8] Jablanović, V. (2011). The chaotic saving growth model: G 7. *Chinese Business Review, 10*(5), pg. 317-327.
[9] Jablanović, V. (2012). *Budget Deficit and Chaotic Economic Growth Models.* Aracne editrice.S.r.l., Roma
[10] Li, T., & Yorke, J. (1975). Period three implies chaos. *American Mathematical Monthly, 8,* pg. 985-992.
[11] Lorenz, E. N. (1963). Deterministic nonperiodic flow. *Journal of Atmospheric Sciences, 20,* pg. 130-141.
[12] Lorenz, H. W. (1993). *Nonlinear dynamical economics and chaotic motion* (2nd ed.). Heidelberg: Springer-Verlag.
[13] May, R. M. (1976). Mathematical models with very complicated dynamics. *Nature, 261,* pg 459-467.
[14] Medio, A. (1993). *Chaotic dynamics: Theory and applications to economics.* Cambridge: Cambridge University Press.
[15] Medio, A. (1996). Chaotic dynamics. Theory and applications to economics, Cambridge University Press, In *De Economist, 144*(4), pg 695-698.

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